Strengths and weaknesses of the logistic function used in forecasting

Theodore Modis[1]

Abstract: This work describes strengths and weaknesses of the logistic function used in forecasting from a theoretical and a practical point of view. Theoretical topics treated are: generalizing the concept of competition, dividing the growth cycle in four "seasons", and using logistics simply qualitatively to obtain rare insights and intuitive understanding. Practical topics addresses are: determination of the uncertainties, how to decide whether to fit cumulative or per unit of time data, and how to deal with a bias toward a low ceiling.

Keywords: logistic growth, S-curve, forecasting, uncertainties, business seasons

[1] Growth Dynamics, Lugano, Switzerland
Address: Theodore Modis, Via Selva 8, 6900 Massagno, Lugano, Switzerland.
Tel. 41-91-9212054, E-mail: tmodis@yahoo.com

1. The natural law of growth in competition: Logistic growth

As early as in 1925 Alfred J. Lotka demonstrated that manmade products diffuse in society along S-shaped patterns similar to those of the populations of biological organisms. (Lotka, 1025). Since then S curve logistic descriptions have made their appearance in a wide range of applications from biology, epidemiology and ecology to industry, competitive substitutions, art, personal achievement and others (Fisher and Pry, 1971; Marchetti, 1983; Meade, 1984; Modis, 1992). The reader is also referred to §2.3.18 and §3.4.5. In fact, logistic growth can be detected whenever there is growth in competition, and competition can be generalized to a high level of abstraction, e.g. diseases competing for victims and all possible accidents competing for the chance to be materialized.

S curves enter as modular components in many intricate natural patterns. One may find S curves inside S curves because logistics portray a fractal aspect; a large S curve can be decomposed in a cascade of smaller ones (Modis, 1994). One may also find chaos by rendering the logistic equation discrete (Modis and Debecker, 1992). Finally, logistics sit in the heart of the Volterra–Lotka equations, which describe the predator–prey relations and other forms of competition. In its full generality, the logistic equation, in a discrete form, with cross terms to account for all interrelations between competing species, would give a complete picture in which growth in competition, chaos, self-organization, complex adaptive systems, autopoiesis, and other such academic formulations, all ensue as special cases (Modis, 1997).

Each S curve has its own life cycle, undergoing good and bad "seasons" see Figure 1. A large set of behaviors have been tabulated, each one best suited for a particular season (Modis, 1998). Becoming conservative—seeking no change—is appropriate in the summer when things work well. But excellence drops in second place during the difficult times of winter—characterized by chaotic fluctuations—when fundamental change must take place. Learning and investing are appropriate for spring, but teaching, tightening the belt, and sowing the seeds for the next season's crop belong in the fall. Focusing on *what* to do is appropriate in spring, whereas in fall the emphasis shifts to the *how*. For example, the evolution of classical music followed a large-timeframe S curve beginning in the fifteenth century and reaching a ceiling in the twentieth century, see Figure 2 (Modis, 2013). In Bach's time composers were concerned with *what* to say. The value of their music is in its architecture and as a consequence it can be interpreted by any instrument, even by simple whistling. But two hundred years later composers such as Debussy wrote music that depends crucially on the interpretation, the *how*. Classical music was still "young" in Bach's time but was getting "old" by Debussy's time. No wonder Chopin is more popular than Bartók. Chopin composed during the "summer" of music's S curve when public preoccupation with music grew fastest. Around that time composers were rewarded more handsomely than today. The innovations they made in music—excursions above the curve—were assimilated by the public within a short period of time because the curve rose steeply and would rapidly catch up with each excursion/innovation. But today the curve has flattened and composers are given limited space. If they make an innovation (excursion above the curve) they find themselves above the curve and there won't be any time in the future when the public will appreciate their work, see Figure 3 (Modis, 2007). On the other hand, if they don't innovate, they will not be saying anything new. In either case today's composers will not be credited with an achievement.

S curves constructed only qualitatively can be accurate, informative, and insightful.

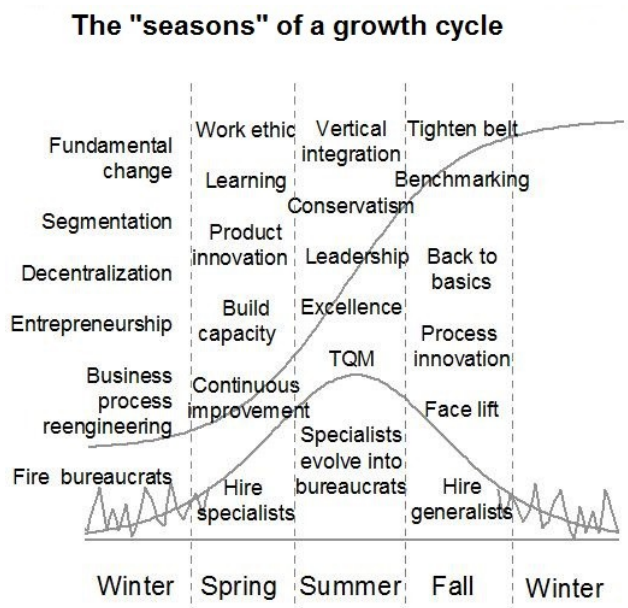

Figure 1. Typical attributes of a growth cycle's "seasons".
Adopted from (Modis, 1998) with permission from the author.

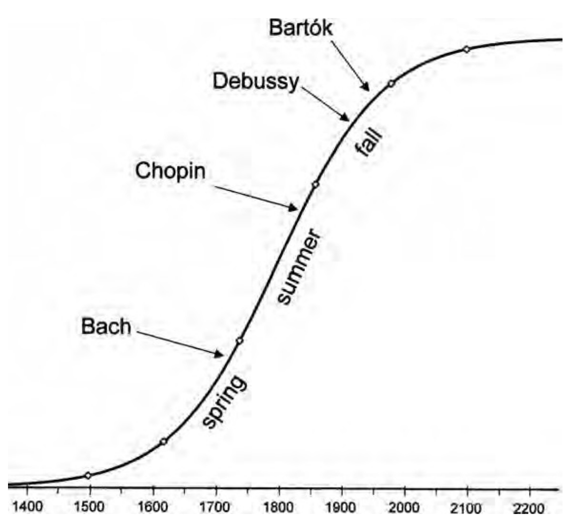

Figure 2. The vertical axis could be something like "importance of music", "public appreciation of music", or "public preoccupation with music", (always cumulative). Adopted from (Modis, 2013) with permission from the author.

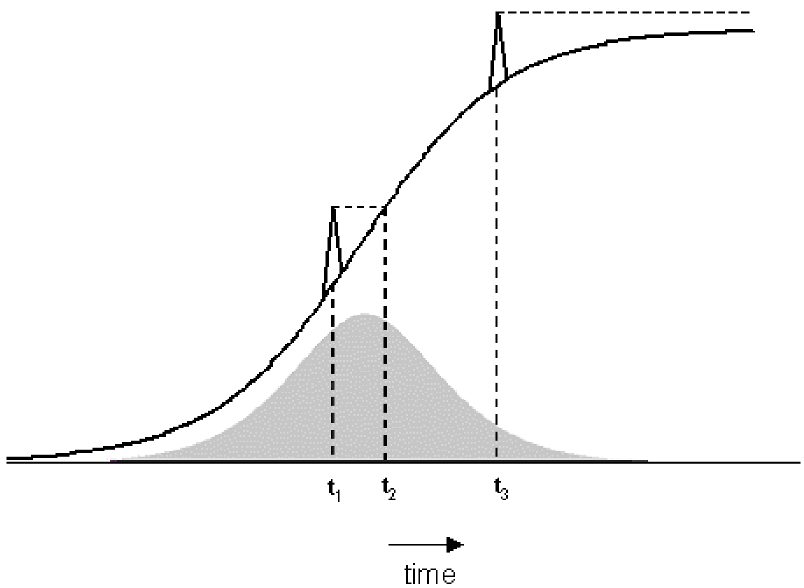

Figure 3. An upward excursion at t_1 reaches the same level as the logistic curve at t_2 and can be considered as a "natural" deviation. The same-size excursion at time t_3 has no corresponding point on the curve. The gray life cycle delimits the position and size of all "natural" deviations. Adopted from (Modis, 2007) with permission from the author.

2. Dealing with logistic forecasts in practice

The most fascinating aspect of S-curve fitting is the ability to predict from early measurements the final ceiling. This very fact, however, constitutes also the fundamental weakness and the major criticism of these forecasts because logistic fits on early data can often accommodate very different values for the final ceiling. Debecker and Modis have carried out an extensive simulation study aiming to quantify the uncertainties on the parameters determined by logistic fits (Debecker and Modis, 1994.) The study was based on 35,000 S-curve fits on simulated data, smeared by random noise and covering a variety of conditions. The fits were carried out via a χ^2 minimization technique. The study produced lookup tables and graphs for determining the uncertainties expected on the three parameters of the logistic function.

A frequent point of misunderstanding and confusion is whether a forecaster should fit an S-curve to the cumulative number or to the number per unit of time. Here the forecaster must exercise wise judgment. What is the "species" and what is the niche that is being filled? To the frustration of business people there is no universal answer. When forecasting the sales of a new product it is often clear that one should fit the cumulative sales because the product's market niche is expected to eventually fill up. But if we are dealing with something that is going to stay with us for a long time (e.g. the Internet or a smoking habit), then one should not fit cumulative numbers. At times this distinction may not be so obvious. For example at the appearance of COVID-19 many people (often amateurs) began fitting S-curves to the cumulative number of infections. Some of them were rewarded because indeed the diffusion of the virus in some countries behaved accordingly (Debecker and Modis, 2021.) But many were frustrated and tried to "fix" the logistic equation by introducing more parameters, or simply gave up on trying to use logistics with COVID-19. And yet, there are special situations (e.g. the US), which can be illuminated by logistic fits but on the daily number of infections, not on the cumulative number. As of August 1, 2020, leaving out the three eastern states that had gotten things under control, the rest of the US displayed two classic S-curve steps followed by plateaus, see Figure 1. The two plateaus reflect the number of infections that American society was willing to tolerate at the time, as the price to pay for not applying measures to restrict the virus diffusion.

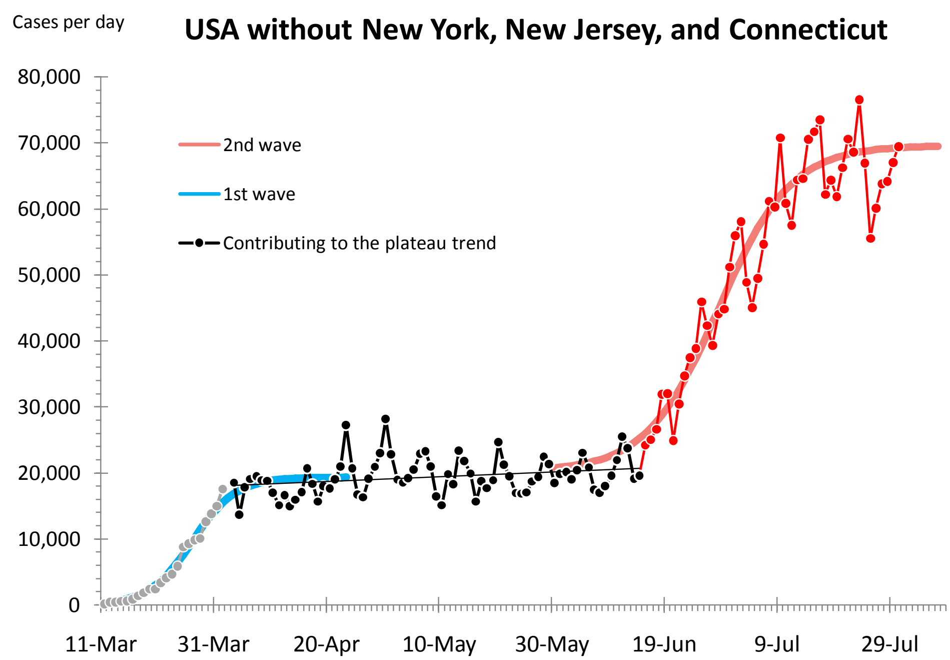

Figure 1. Two logistic-growth steps during the early diffusion of COVID-19 in America.

No matter what fitting program one uses, the fitted S-curve will flatten toward a ceiling as early and as low as it is allowed by the constraints of the procedure. As a consequence fitting programs may yield logistic fits that are often biased toward a low ceiling. Bigger uncertainties on the data points accentuate this bias by permitting larger margins for the determination of the S-curve parameters. To compensate for this bias the user must explore several fits with different weights on the data points during the calculation of the χ^2. He or she should then keep the answer that gives the highest ceiling for the S-curve (most often obtained by weighting more heavily the recent historical data points). Of course, this must be done with good justification; here again the forecaster must exercise wise judgment.

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