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Chapter 3

INSIGHTS ON COMPETITION FROM A SCIENCE-BASED ANALYSIS

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ABSTRACT

The work presented here uses a science-based approach to obtain new understandings on the mechanisms and the ramifications of competition in everyday life. Assuming competition of a Darwinian nature we can deduce an S-shaped pattern for growth in most competitive environments. Examples range from a rabbit population growing in a fenced-off grass field to scientists competing for Nobel-Prize awards. There are secrets embedded in the mathematical law that describes growth in competition. The rate of growth being proportional to the amount of growth already achieved makes beginnings difficult and sheds light on such proverbial wisdom as "you need goal to make gold". It also argues for the necessity to engage teachers in the learning process. Other revelations are linked to the symmetry of a life-cycle pattern, which possesses predictive power and demystifies the easy-come-easy-go phenomenon. Predictive power characterizes the rapid-growth phase of the S-shaped pattern (rheostasis) as well as the end of the pattern when growth reaches a ceiling (homeostasis) where supply and demand are in equilibrium. The latter phenomenon is best exemplified by society's tolerance of deadly car accidents because deaths from car accidents have remained at an invariant level for many decades reflecting equilibrium. The mathematical equation for growth in competition when cast in discrete form reveals fluctuations of chaotic nature before and after the rapid-growth phase. This can illuminate the turbulent times before and after the formation of the USSR as well as the tumultuous times of the 1930s in America. Extending the quantitative approach to two species competing in the same niche involves introducing coupling constants that account for how one species impacts the growth rate of the other. A celebrated example is the predator-prey relationship, which is only one of six possible interactions all of which can be encountered in the marketplace where products and companies compete like species. There are six possible dimensions for action in a two-species competitive struggle that can be exploited toward managing

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competition and setting one's role/image in the marketplace. An example dealt in detail is the evolution of the number of American Noble-Prize winners whose numbers are not about to begin diminishing. Americans are involved in a win-win competitive struggle with non-American scholars, but Americans are drawing more of a benefit.

INSIGHTS ON COMPETITION FROM A SCIENCE-BASED ANALYSIS

The fisherman starting his day off the Adriatic coast was wondering whether it was going to be a day of big fish or smaller ones. He had seen this phenomenon often. He would start by catching a big fish in the morning and then for the rest of the day it would be one big fish after another. Other times it would be small catches all day long. He was reminded in passing of a biblical reference to periods of fat years and thin years, but he got down to work without further philosophizing. He had no time to waste; in the days following the Great War the sea was one place where food could still be found relatively easily.

Meanwhile, at the University of Siena, the biologist Umberto D'Ancona was making statistical studies of Adriatic fisheries. He found temporary increases in the relative frequency of the more voracious kinds of fish, as compared with the fish on which they preyed. Vito Volterra, a mathematician at the University of Rome, was preoccupied in his own way with the same phenomenon. He knew of D'Ancona's observations and believed he understood the reason for them. Since big fish eat small fish — and consequently depend on them for survival — some interchange in population dominance should be expected. The population of big fish would grow until small fish became scarce. At some point the big fish would starve and their diminishing numbers would give the small-fish survivors a chance to renew their numbers. Could this phenomenon be described mathematically?

Volterra succeeded in building a mathematical formulation that described well the fisherman's observations. A model for the growth of populations, it states that the rate of growth is limited by competition and that the overall size of the population (for example, the number of rabbits in a fenced-off grass field) slowly approaches a ceiling, the height of which reflects the capacity of the ecological niche. The model would serve as a foundation for modern biological studies of the competitive struggle for life. Alfred J. Lotka also studied such problems to some extent. Today, there are applications bearing both men's names.

Half a century later, Cesare Marchetti, a physicist at the International Institute of Applied Systems Analysis (IIASA) near Vienna, Austria, was given the task by the energy-project leader to forecast energy demands. Another kind of war had been shaking the West recently: the fierce competition for oil. The need for increased understanding of the future energy picture was becoming imperative. Marchetti approached the problem as a physicist, who sought answers through the use of the scientific method: observation, prediction, verification. In this approach predictions must be related to observations through a theory resting on hypotheses. When the predictions are verified, the hypotheses become laws. The simpler a law, the more fundamental it is and the wider its range of applications.

Marchetti had long been concerned with the "science" of predictions. In his work, he first started searching for what physicists call invariants. These are constants universally valid and manifested through indicators that do not change over time. He believed that such indicators represent some kind of equilibrium even if one is not dealing with physics but with human activities instead. He then suspected that the fundamental laws which govern growth and competition among species may also describe human activities. Competition in the market place can be as fierce as in the jungle, and the law of the survival of the fittest becomes indisputable. Marchetti noted that growth curves for animal populations follow patterns similar to those for product sales. Could it be that the mathematics developed by Volterra for the growth of a rabbit population describe equally well the growth of cars and computers? Marchetti went on to make a dazzling array of predictions, including forecasts of future energy demands, using Volterra's equations.[1] But how far can the analogy between natural laws and human activities be pushed and how trustworthy are the quantitative forecasts based on such formulations?

In 1984 my professional lifeline crossed those of Volterra and Marchetti. I was passing from academia to industry, leaving fifteen years of research in elementary particle physics to work as a management-science consultant for DEC (Digital Equipment Corporation). My boss, an ex-physicist himself, tried to smooth the transition by showing me some of Marchetti's papers that described applications of laws from the natural sciences to a variety of human affairs. "See how we are also intellectually alert in industry" was the message. However, three weeks later, and in spite of my enthusiasm, the stern new message was: "Now leave all this aside and let's get down to work." It was too late, because the subject had intrigued me.

From then onward my involvement with natural growth in competition became my *raison d'être* and culminated with the publication of my first book *Predictions – Society's Telltale Signature Reveals the past and Forecasts the Future* (Modis, 1992), which begins the same way as this chapter.

THE S-CURVE

At the heart of competition lies the principle of survival of the fittest. If you put a pair of rabbits in a meadow and the average rabbit litter is taken as two, you can watch the rabbit population go through the successive stages of 2, 4, 8, 16, 32, 64, ..., 2^n in an exponential growth. There is a population explosion up to the time when a sizable part of the ecological niche is occupied. It is only after this time that limited food resources begin imposing constraints on the number of surviving rabbits and the population growth slows down as it approaches a ceiling — the capacity of a species' ecological niche. This slowdown may happen by means of increased kit mortality, diseases, lethal fights between overcrowded rabbits, or even other more subtle forms of behavior that rabbits may act out unsuspectingly. Nature imposes population controls as needed, and in a competitive environment, only the fittest survive.

Over time, the rabbit population traces an S-shaped trajectory. The *rate* of growth traces a curve that is bell-shaped and peaks when half the niche is filled. The S-shaped curve (S-curve) for the population and the bell-shaped curve for its rate of growth constitute a pictorial representation of the natural growth process — that is, how a species population grows into a limited space by obeying the law of survival of the fittest.

At the ceiling, we may witness oscillations as the rabbit population explores the possibility to go further and overshoots the niche capacity only to fall back later giving the

1 See Marchetti's website: http://www.cesaremarchetti.org

grass a chance to grow back and feed more rabbits. At this point we may talk of a *homeostasis*, a stable state of equilibrium between the number of rabbits and the amount of grass.

The Patterns of Natural Growth in Competition and Its Life Cycle

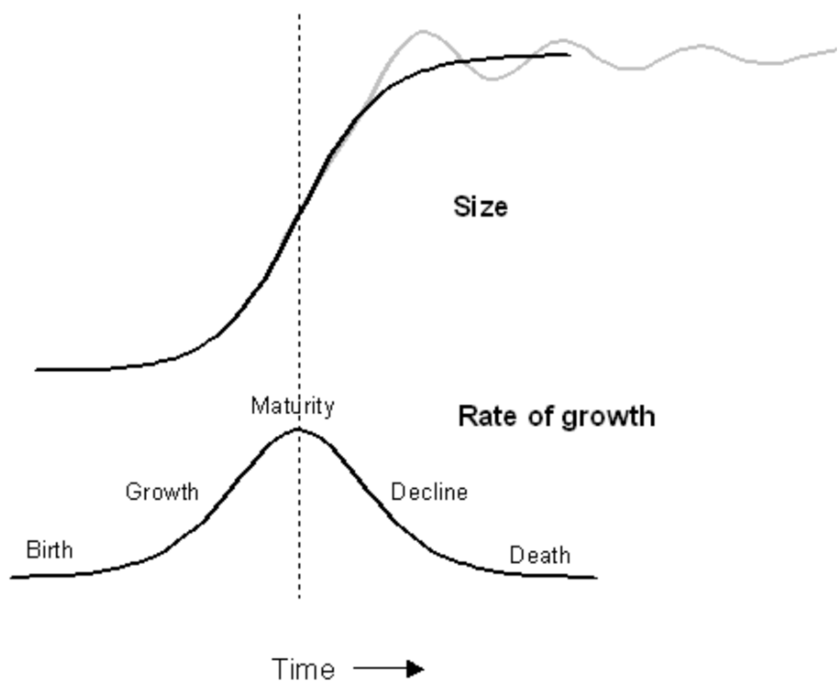

Figure 1. The S-shaped pattern above is the solution of the natural-growth equation. The bell-shaped curve below is used across disciplines as a template for the cycle of life. The gray line sketches the fluctuations of a rabbit population that has filled its ecological niche to capacity.

An S-curve and the associated life cycle are two different ways of looking at the same growth process. The S-curve represents the size of the growth and points out (anticipates) the growth potential, the level of the final ceiling, how much could one expect to accomplish. The bell-shaped life-cycle curve represents the rate of growth and is more helpful when it comes to appreciating the growth phase you are traversing, and how far you are from the end. The S-shaped curve reminds us of the fact that competitive growth is capped. The bell-shaped curve reminds us that whatever gets born eventually dies. From an intuitive point of view, an S-curve promises a certain amount of growth that can be accomplished, whereas a bell-curve heralds the coming end of the process as a whole. Both curves possess predictive power.

At the ceiling of the S-curve (homeostasis) the level remains invariant and therefore it is trivial to forecast. But there is predictability also during the rapid-growth phase (rheostasis). You can easily anticipate where a fast-moving train will end up. A bicycle is stable only when in motion and the faster it is going the more stable it is, the easier it is to project its trajectory.

The predictive power of the bell-shaped life-cycle curve comes from its symmetry. A rapid rise will be followed by an equally rapid decline, echoing such expressions as “Easy come, easy go” and “Early ripe, early rot”. Many business endeavors have experienced this the hard way in the marketplace.

The mathematical equation — the logistic equation — that describes the law of natural growth in competition and gives rise to the S-curve says in words that the rate of growth must be at all times proportional to two things:

- The amount of growth *already accomplished*.
- The amount of growth *remaining to be accomplished*.

If either one of these quantities is small, the rate of growth will be small. This is the case at the beginning and at the end of the process. The rate is greatest in the middle, where both the growth accomplished and the growth remaining are sizable. Furthermore, growth “remaining to be accomplished” implies a limit, a saturation level, a finite niche size. Competition is a consequence of a limited resource and therefore growth in competition cannot go on forever; it is necessarily capped. This ceiling of growth is assumed to be constant throughout the growth process. Such an assumption is a good approximation to many natural-growth processes, for example, plant growth, in which the final height is genetically pre-coded.

It is a remarkably simple and fundamental law. Besides used by biologists to describe species populations, it has also been used in medicine to describe the diffusion of epidemic diseases. J. C. Fisher and R. H. Pry refer to the logistic equation as a diffusion model and use it to quantify the spreading of new technologies into society (Fisher & Pry, 1971). One can immediately see how ideas or rumors may spread according to this law. Whether it is ideas, rumors, technologies, or diseases, the rate of new occurrences will always be proportional to how many people have it and to how many don’t yet have it. At the end you will always be able to find — albeit in slowly diminishing numbers — the outcasts who never heard the rumor, or refused to adopt the new technology.

The S-curve has also being referred to as a learning curve in psychology as well as in industry. For example, the evolution of an infant’s vocabulary has been shown to follow an S-curve that reaches a ceiling of about 2500 words by age six.[2] Acquiring vocabulary can be thought of as a competitive process where words in the combined active vocabulary of the two parents compete for the infant’s attention. The words most frequently used will be learned first, but the rate of learning will eventually slow down because there are fewer words left to learn. This ceiling of 2500 words defines the size of the home vocabulary “niche,” all the words available at home. Later, of course, schooling enriches the child’s vocabulary, but this is a new process, starting another cycle, following probably a similar type of curve to reach a higher plateau.

The S-curve in Figure 1 is asymptotic, i.e. it approaches zero, the level of the ceiling continuously but reaches it only in time $-\infty$, $+\infty$ respectively. On the other hand the fact that growth is proportional to the amount of growth already achieved renders the beginning of every natural-growth process practically very difficult (theoretically impossible because zero growth achieved yields a null rate for growth and so things cannot be started!) This

2 An S-curve has been fitted on the data found in Whiston (1974).

demystifies the known difficulty associated with beginnings. An ancient Greek proverb on achievement equates the beginning with half of the whole! The consequences on learning are enlightening. Theoretically learning cannot begin without outside help. The work of teachers becomes indispensable in this context. The teacher is the custodian of knowledge and oriental schools of thought preclude search for esoteric knowledge and personal development without a teacher.

The theoretical difficulty in getting growth in competition started touches upon philosophical questions akin to the genesis because of the requirement that some discontinuous intervention from an external agent (for example, a powerful intelligent entity) is necessary in order to get something going from nothing.

Fatal Car Accidents

The logistic equation has been successfully used to describe growth processes where the notion of competition has been raised to remarkable levels of abstraction. Marchetti has argued that primary-energy sources compete for consumers' favor and diseases compete for victims. In all cases there is a limited resource, which imposes the constraint that only the best-fit candidate wins. My favorite example is fatal car accidents (Marchetti, 1983). All possible accidents can be thought to compete for becoming realized and claim victims. Only the "best" of them will do so because here again there is a limited resource and contrary to what one may naively expect it is much smaller than the entire population.

Car safety has been a passionate subject frequently appearing in headlines. At some point in time cars had been compared to murder weapons. Still today close to two hundred thousand people worldwide die from car accidents every year, and up to ten times as many suffer injuries. Efforts are continually made to render cars safer and drivers more cautious. How successful have such efforts been? Can this rate be significantly reduced as we move toward a more advanced society?

To answer these questions, we must look at the history of car accidents, but in order to search for a fundamental law we must have accurate data and an appropriate indicator. Deaths are recorded and interpreted with less ambiguity than other accidents. Moreover, the car as a public menace is a threat to society, which may "feel" the pain and react accordingly. Consequently, the number of deaths per one hundred thousand inhabitants per year becomes a better indicator than accidents per mile, or per car, or per hour of driving.

The data shown in Figure 2 are for the United States starting at the beginning of the 20th century. What we observe is that deaths caused by car accidents grew along an S-shaped pattern with the appearance of cars until the mid 1920s, when they reached about twenty-four per one hundred thousand per year. From then onward they seem to have stabilized, even though the number of cars continued to grow. A homeostatic mechanism seems to emerge when this limit is reached, resulting in an oscillating pattern around the equilibrium position. The peaks may have produced public outcries for safety, while the valleys could have contributed to the relaxation of speed limits and safety regulations. What is remarkable is that for over sixty years there has been a persistent self-regulation on car safety despite major increases in car numbers and performance, and important changes in speed limits, safety technology, driving legislation, and education.

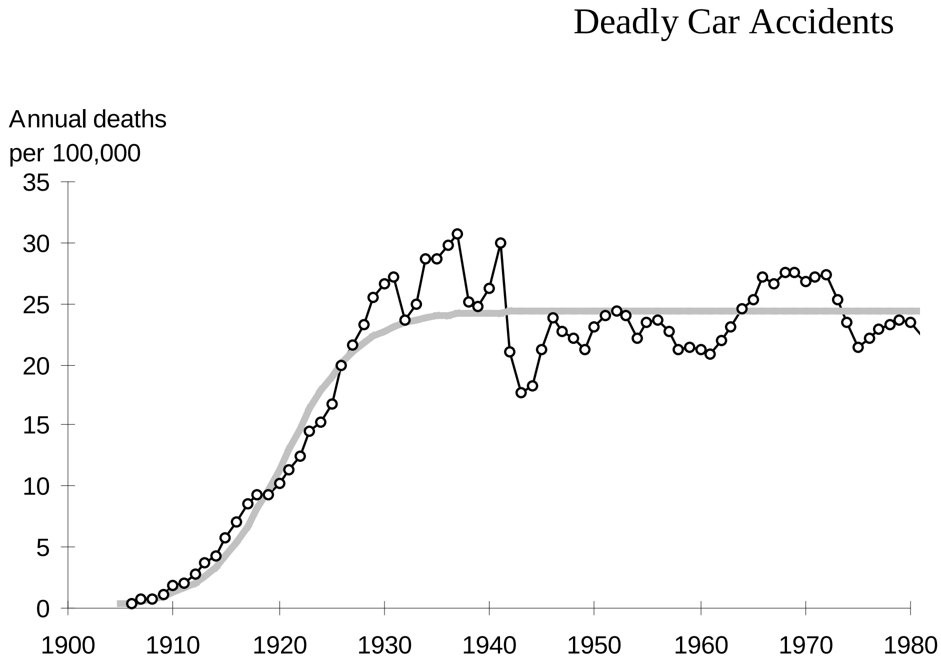

Figure 2. The annual number of deaths from motor-vehicle accidents per 100,000 population has followed an S-curve to reach a ceiling of 24 around which it has been fluctuating since the mid 1920s, not unlike the rabbit population sketched in gray in Figure 1. The peak in the late 1960s provoked a public outcry that resulted in legislation making seat belts mandatory.

Why the number of deaths is maintained constant and how society can detect excursions away from this level? Is it conceivable that some day car safety will improve so much that car accidents will be reduced to zero? American society has tolerated this level of accidents for more than half a century. A Rand analyst has described it as follows: “I am sure that there is, in effect, a desirable level of automobile accidents — desirable, that is, from a broad point of view, in the sense that it is a necessary concomitant of things of greater value to society” (Williams, 1958). Abolishing cars from the roads would certainly eliminate car accidents, but at the same time it would introduce more serious hardship to citizens.

An invariant (a homeostatic level) can be thought of as a state of well-being. It has its roots in nature, which develops ways of maintaining it. Individuals may come forward from time to time as advocates of an apparently well-justified cause. What they do not suspect is that they may be acting as agents to deeply rooted forces maintaining a balance that would have been maintained in any case. An example is Ralph Nader’s crusade for car safety, *Unsafe at Any Speed*, published in the 1960s, by which time the number of fatal car accidents had already demonstrated a forty-year-long period of relative stability. But examining Figure 2 more closely, we see that the 1960s show a small peak in accidents, which must have been what prompted Nader to blow the whistle. Had he not done it, someone else would have. Alternatively, a timely social mechanism might have produced the same result; for example, an “accidental” discovery of an effective new car-safety feature.

During the last thirty years there has been evidence for a gentle downward trend not shown in Figure 2. One could argue that Nader’s crusade for car safety was indeed effective. After all it was instrumental in making seat belts mandatory and lowering the speed limits throughout the country. I seriously doubt such cause-and-effect reasoning. Seat belts and speed limits certainly had some effect, which among other things, made environmentalists shift their focus to other issues. But action taken forty years ago would not still keep reducing deaths from car accidents today. In fact speed limits have been significantly raised in most states since then. The state of Montana has even experimented with lifting some speed limits altogether.

As usually, there is a more deeply seated explanation for deviations from a natural-growth pattern. The airplane has been steadily replacing the automobile as a means of intercity transportation since the late 1960s. Despite the fact that the automobile still commands a dominant share of the transportation market today, Americans have in fact been giving up, slowly but steadily, their beloved cars, and the fatal accidents that go with them.

PERSONAL ACHIEVEMENT

Competition is abundant in the worlds of the arts and the sciences. But there is one aspect of competition that we are not familiar with. The fact that one’s creative potential is finite, which results in a competitive squeeze for the realization of one’s remaining creative impulses.

It was Marchetti again who first associated the evolution of a person’s creativity and productivity with natural growth. He assumed that a work of art or science is the final expression of a “pulse of action” that originates somewhere in the depths of the brain and works its way through many intermediate stages to produce a creation. He then studied the number of these creations over time and found that their growth follows S-shaped curves. Each curve presupposed a final ceiling, a niche size, a perceived creative potential. “Perceived” because competition may prevent it from being reached. Marchetti proceeded to study hundreds of well-documented artists and scientists. In each case, he took the total number of known creations, graphed them over time, and determined the S-shaped curve that would best connect these data points. He found that most people died close to having realized their perceived potential. In his words:

"To illustrate further what I mean ... consider the amount of beans a man has in his bag and the amount left when he finally dies. Looking at the cases mentioned here ... I find that the leftover beans are usually five to ten percent of the total. Apparently when Mozart died at 35 years of age, he had already said what he had to say" (Marchetti, 1985).

The idea is intriguing. Obviously people’s productivity increases and decreases with time. Youngsters cannot produce much because they have to learn first. Old people may become exhausted of ideas, energy, and motivation. It makes intuitive sense that productivity goes through a life cycle over a person’s lifetime, slowing down as it approaches the end. The cumulative productivity — the total number of works produced — could very well look like an S-shaped curve over time.

So I looked up Mozart’s compositions and was able to fit an S-curve on the evolution of his work volume, see Figure 3. I counted every composition as one unit, on the argument that a minuet at the age of six is no less a creative achievement than a requiem at the age of thirty-five. The fit turned out to be successful. I found an S-curve that passed impressively close to all thirty-one yearly points representing the cumulative number of compositions. There were two little irregularities, however; one on each end.

The irregularity at the low end of the curve caused my computer program to include an early-missing-data parameter. The reason: better agreement between the curve and the data if 18 compositions are assumed to be missing during Mozart's earliest years. His first recorded composition was created in 1762, when he was six. However, the curve's nominal beginning — the 1 percent level of the ceiling — is around 1756, Mozart's birth date. Conclusion: Mozart was composing from the moment he was born. His first eighteen compositions, however, were never recorded due to "technical" difficulties — the fact that he could neither write nor speak well enough to dictate them to his father.

Mozart (1756 – 1791)

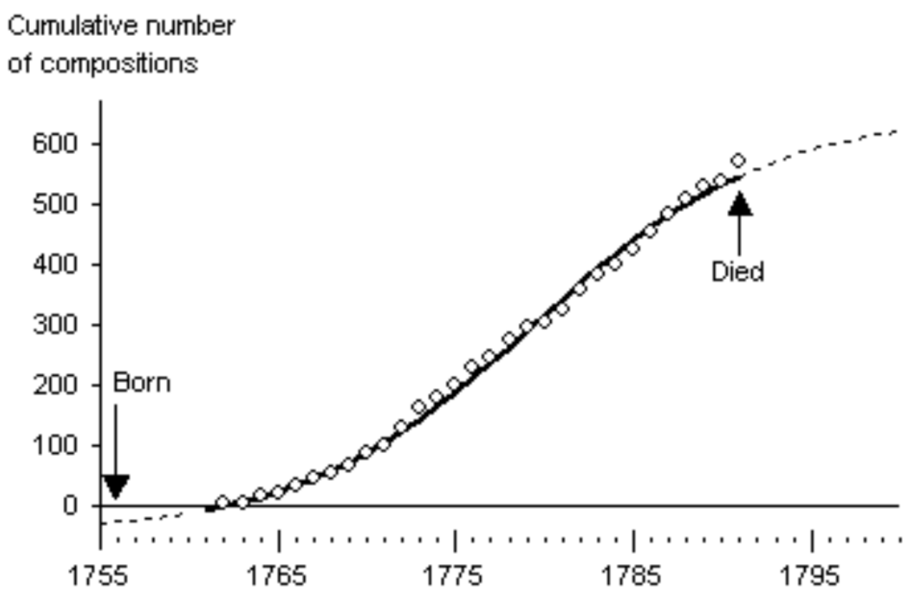

Figure 3. The best-fitting S-shaped curve implies 18 compositions "missing" between 1756 and 1762. The nominal beginning of the curve — the 1 percent level — points at Mozart's birthday. The nominal end — the 99 percent level — indicates a potential of 644 works.

The second irregularity was at the high end of the curve, the year of Mozart's death: 1791 showed a large increase in productivity. In fact, the data point fell well above the curve, corresponding more to the productivity projected for the year 1793. What was Mozart trying to do during the last year of his life? With his creative potential determined as 644 compositions, his last composition would put him at the 91 percent level of exhaustion. Most people who die of old age have realized 90 percent of their creative potential. There was very little left for Mozart to do. His work in this world had been practically accomplished. The irregularity at the high end of his creativity curve indicated the sprint at the finish! What he had left to do was not enough to help him fight the illness that was consuming him. "Mozart died of old age" is the conclusion we would come to by looking at this graph. Yet there is a popular belief that the world has been deprived of many musical masterpieces by his "premature" death.

In discussions with musicians, I have found that many are not shocked by the idea that Mozart may have exhausted his creative potential at the age of thirty-five. He had already contributed so much in every musical form of the time that he could no longer break new ground. Of course, he could have done more of the same: more concertos, more symphonies, more trios and quartets. But all this would have represented com-promised innovation. He himself wrote at the age of 21, "To live until one can no longer contribute anything new to music" (Massin, 1978).

From Order to Chaos and Back

When the logistic equation that describes growth in competition is cast into discrete form (necessary because everything in real life is discrete) it becomes the chaos equation. The latter is strikingly similar to the former — see Appendix — but whereas the former gives rise to the smooth S-curves, the latter, for certain values of its parameters, gives rise to states of chaos. The logistic equation emphasizes the presence of a trend and has become the tool to describe natural growth. The chaos equation emphasizes the lack of trend and has become the tool to describe chaotic states. Both equations originate with growth in competition of Darwinian nature.

The states of chaos appear on what corresponds to the ceiling of the logistic after the upward trend has died down. It has also been shown that chaotic-type fluctuations could be expected before as well as after the curve's steep rise (Modis & Debecker, 1992). An example in a large time frame is the world economy, as evidenced by the evolution of energy consumption. Per-capita energy consumption worldwide is more than seven times greater today than it was 150 years ago. This increase took place, not in a steady, uniform rate, or even in a random fashion, but in two well-defined S-shaped steps.

It is easy to see why energy consumption is a competitive process. Human appetite for energy is insatiable — they will use up all the energy they can get — but supply is limited because the procurement of energy is difficult (read expensive). From time to time technology and socioeconomic conditions permit/stimulate the opening up of new energy-supply niches. When this happens energy consumption increases to exhaust these niches in a natural way, namely along S-shaped patterns. At the end of the growth step energy consumption reaches a homeostasis; further growth is held back by other more urgent priorities.

In Figure 4 we see that the first step ended around 1920 with a period of stagnation that lasted for about two decades. The second energy consumption step was completed around 1975, and we have just witnessed the beginning of a third step. There can be little doubt that this indicator will go through another growth phase considering the dire need for industrial growth in the developing world.

Energy consumption correlates in an unambiguous way with industrial development and economic prosperity. The profile of the energy curve over time eloquently points out two chaotic low-growth periods, one centered on the mid-1930s and another one around 1990. These economic depressions echo Kondratieff's economic cycle (Kondratieff, 1935). Competition intensifies as we enter these periods. Remember the rabbits, they began feeling the squeeze when their ecological niche began filling up.

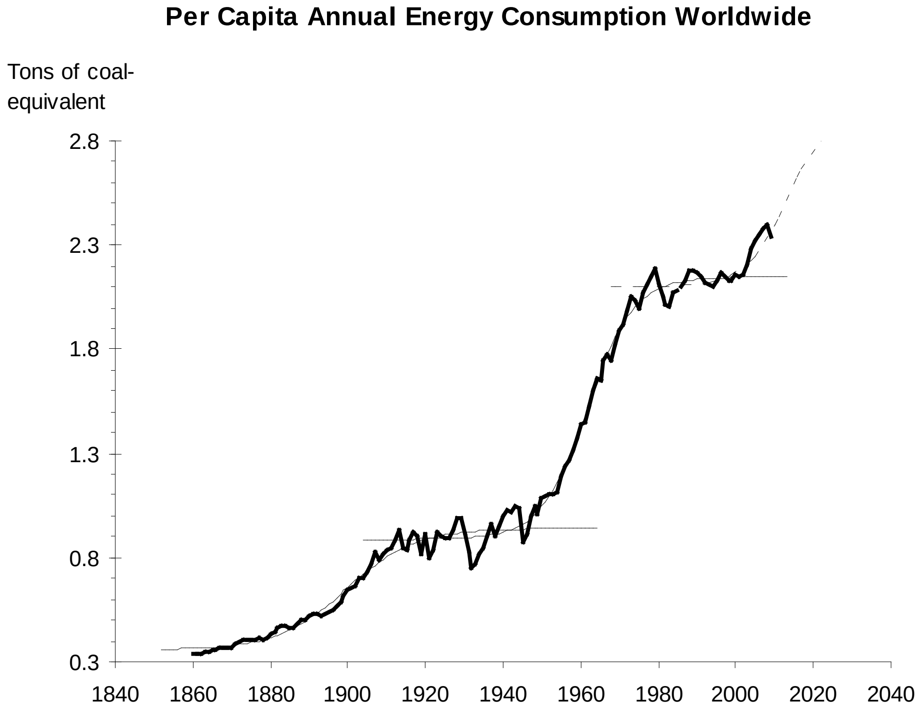

Figure 4. The data display sustained growth in terms of a succession of S-shaped steps. The two smooth solid lines are logistic fits to the data. The intermittent line is a scenario for the future suggested by analogy. The data pattern exemplifies the alternation between orderly growth and turbulent stagnation periods.

The Rise and Fall of the Communist Empire

The ultimate sin of a forecaster is to predict something after it has happened. Well, my work revolves around innovative forecasting techniques, and here I am about to present arguments demonstrating that the collapse of Communism could have been predicted with devilish precision as early as the 1960s. What makes me indulge in such an unprofessional exercise is the fact that recently one more person threw at me the by now classic remark: “Who could have ever predicted the fall of the Berlin wall?” His comment was the last straw.

We saw earlier that chaotic periods precede and follow the rapid-growth phase of a natural-growth process. At the same time, life cycles are generally symmetric. Communism and the USSR can be likened to a species whose life cycle began with the revolution of 1917. It peaked forty years later, in the mid-1950s, when the Soviet Union successfully competed and often surpassed the United States (for example, with Sputnik in 1957). A symmetric life cycle would position the end of communism another forty years later, the mid-1990s. The headline-making event, the collapse of the Berlin wall, took place in a sharp discontinuous way, reflecting the first large fluctuation of the chaotic state that sets in as the natural-growth process approaches completion. With the end of the process anticipated in the mid-1990s, chaotic tremors are to be expected several years earlier. These are rather accurate predictions for the collapse of communism and the fall of the Berlin wall. They ensue from a rigorous and precise reckoning. They could have been made as early as the 1960s, when it became clear that the Soviet Union was already over its peak. The Soviets had begun losing, first in 1963 with the Cuban missile crisis, then in 1969 with the moon race that was doomed for lack of funds. The Soviets could not afford adequate testing of their superior rocket, and it exploded during the critical launch.

The Soviet Union Life Cycle

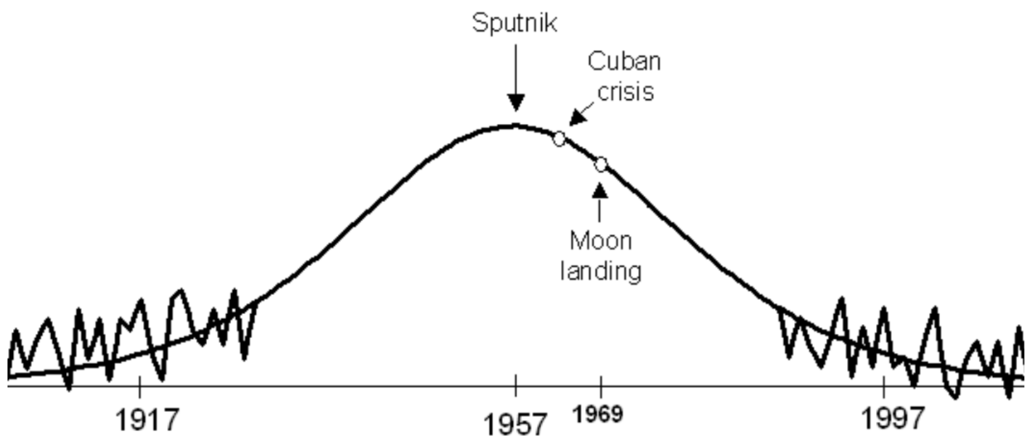

Figure 5. A pictorial representation of the making and the dismantling of the Soviet Union. Chaotic periods characterize the revolution of 1917 as well as the years following the fall of the Berlin wall.

The fall of the Berlin wall punctuated the end of the Communist growth curve and signaled the beginning of the chaotic phase. It should take a while for free market forces to become established there, however. If we divide the life cycle in four equal segments — according to the season metaphor, see *Conquering Uncertainty* — 20 years (one-fourth of the 80-year life cycle of the Soviet block) is how long the post-Berlin-wall chaotic period should last (Modis, 1998).

COMPETITION MANAGEMENT

Extending the quantitative study of competition to more than one species requires the introduction of coupling constants. If there are two species in the niche, then two coupling constants are required to account for how one's presence impacts the growth rate of the other. The mathematical description then consists of two logistic equations each one augmented by one coupling parameters — see Appendix. The set of these equations are referred to as the Volterra-Lotka coupled equations. The usefulness of this formulation has been extended to describe competition outside biology and ecology. Indeed, the Volterra-Lotka model has opened the way to effectively managing competition in the marketplace. A set of elementary marketing actions has emerged that provide guidance when searching for a commercial image or an effective advertising message.

An intriguing aspect of the marketplace is that the nature of competition can change over time. A technology, company, or product does not need to remain prey to another forever. Competitive roles can be radically altered with technological advances or with the right marketing decisions. External light meters, used for accurate diaphragm and speed setting on photographic cameras, enjoyed a stable, symbiotic (win–win) relationship with cameras for decades. As camera sales grew, so did light-meter sales. But eventually, technological developments enabled camera companies to incorporate light meters into their own boxes. Soon, the whole light-meter industry became prey to the camera industry. Sales of external light meters diminished while sales of cameras enjoyed a boost, and the relationship passed from win–win to predator–prey.

The Battle of the Pens

The struggle between fountain pens and ballpoint pens had a different ending, see Figure 6. The substitution of ballpoint pens for fountain pens as writing instruments went through three distinct stages. Before the appearance of ballpoint pens, fountain-pen sales grew undisturbed along an S-curve to fill the writing-instrument market. They were following an S-shaped curve when the ballpoint technology appeared in 1951. As ballpoint sales picked up, those of fountain pens declined in the period 1951 to 1973. Fountain pens staged a counterattack by radically dropping prices. But that effort failed. Fountain pens kept losing

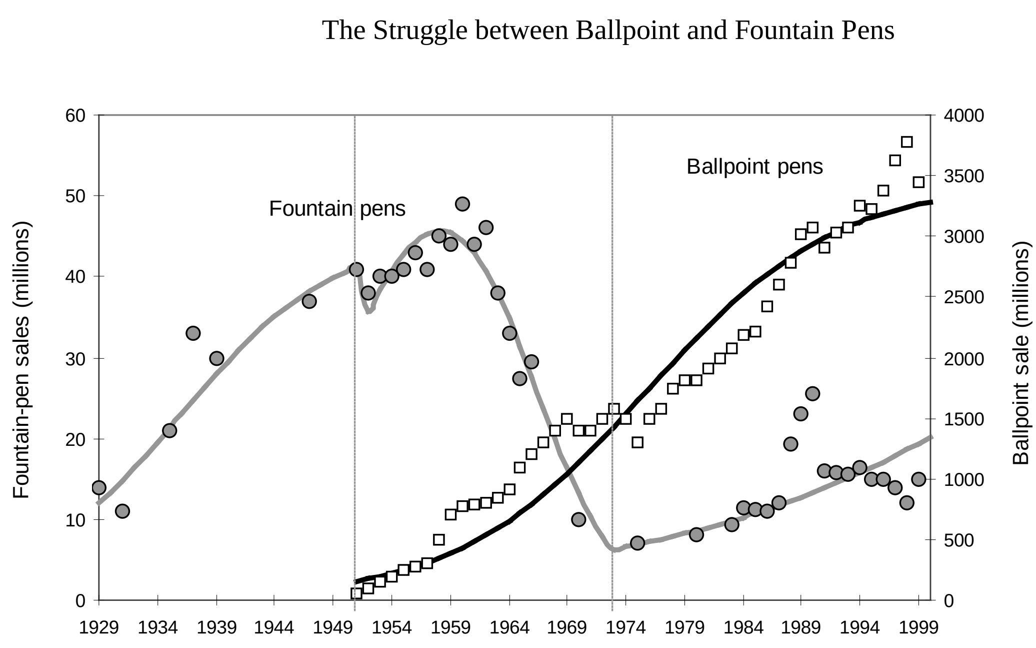

Figure 6. Fountain pen sales were following a classic S-shaped growth curve when ballpoint pens were introduced in 1951. Fountain pens counter attacked by dropping prices between 1951 and 1973, then retreated into noncompetition by entering a luxury niche.[3]

3 Data source: Writing Instrument Manufacturers Association, Mt. Laurel, NL 08054, and Farrell (1993).

market share and embarked on an extinction course. By 1973, their average price had dropped to as low as 72 cents, to no avail.

Eventually, however, the prices of fountain pens began rising. The fountain pen underwent what Darwin would have described as a character displacement to the luxury niche of the executive-pen market. In the early 1970s, the strategy of fountain pens became a retreat into noncompetition. By 1988, the price of some fountain pens in the United States had climbed to $400. The Volterra-Lotka model indicates that today the two species no longer interact but each follows a simple S-shaped growth pattern. As a consequence, fountain pens have secured a healthy and profitable market niche. Had they persisted in their competition with ballpoint pens, they would have perished.

Handling Competition

Character displacement is a classical way to diminish the impact of competition. Another name for this is Darwinian divergence, sometimes also encountered among siblings. In his book *Born to Rebel: Birth Order, Family Dynamics, and Creative Lives*, Frank Sulloway shows that throughout history, first-born children have become conservative and later-borns revolutionaries. First-born children end up conservative because they do not want to lose any of the only-child privileges they enjoy. But this forces later-borns into becoming rebellious, to differentiate themselves and thus minimize competition with a sibling and optimize survival in the same family (Sulloway, 1996).

The attack of a new species against the defenses of an incumbent one lies at the heart of corporate marketing strategies. Christopher Farrell, director of scientific affairs at Baxter Healthcare Corp. (Deerfield, IL), defined an attacker's advantage and a defender's counterattack in terms of the coupling parameters in the Volterrra-Lotka model. A coupling parameter can be determined by data, and thus, it can assign a precise number to an attacker's advantage or a defender's counterattack. The attacker's advantage quantifies the extent to which the attacker inhibits the ability of the defender to keep market share. The defender's counterattack quantifies the extent to which the defender can prevent the attacker from stealing market share (Farrell, 1993).

Under attack, the defender redoubles its efforts to maintain or improve its position. A high value for the defender's counterattack implies a face-on counterattack within the context "what they do, we do better." Kristina Smitalova and Stefan Sujan studied and classified the various coupling schemes by which two competitors might interact. They distinguished and labeled six ways in which two competitors can influence each other's growth rate, according to the sign of the two coupling parameters, Table 1 (Smitalova & Sujan, 1991). Pure competition occurs between rabbits and sheep. Each one diminishes the growth of the other but not necessarily with the same importance (sheep multiply more slowly but eat more). Market examples are the competition among mobile-telephone companies and among different-size computer models.

An example of predator–prey interaction is the case of cinema and television. The more movies made, the more television benefits; but the more television grows in importance, the more cinema suffers. Films made for TV are not shown in movie theaters. Without the legal protection that restricts permission to broadcast new movies, television would probably "eat up" the cinema audience.

Table 1. The six ways two competitors, A and B, can influence each other's growth rate can be summarized in terms of positive, negative, and neutral coupling parameters.

MODE	DEFINITION	Coupling parameter	
		A	B
Pure competition	Both species suffer from each other's existence.	-	-
Predator-prey	One serves as food for the other.	+	-
Mutualism	Symbiosis; a win-win situation.	+	+
Commensalism	A parasitic type of relationship in which one benefits from the existence of the other, which nevertheless remains unaffected.	+	0
Amensalism	One suffers from the existence of the other, which remains impervious to what is happening.	-	0
Neutralism	No interaction whatsoever.	0	0

A typical case of mutualism is software and hardware. Sales of each trigger more sales for the other, as in the early relationship between external light meters and cameras. Add-ons and accessories such as vehicle extras illustrate commensalism. The more automobiles sold, the more car accessories will be sold. The inverse is not true, however; sales of accessories do not trigger automobile sales.

Amensalism can be found with ballpoint pens and fountain pens. The onslaught of ballpoint sales seriously damaged fountain pen sales, yet the ballpoint-pen population grew as if there were no competition.

Neutralism arises in all situations in which there is no market overlap, as happens between fountain pens and ballpoint pens today. Another example is a sports store that sells both swimwear and skiwear. Although sales of one may rise when sales of the other go down because of seasonal variation, sales of one product do not generally affect sales of the other (Modis, 1998).

Coupling Parameters

The S-shaped pattern evidenced in the evolution of a species population can in general be described with two parameters: one reflects the ability of the species to multiply (or a product's attractiveness), and the other reflects the size of the ecological niche (or a product's market niche). But what happens if more than one species of competitor is present? Besides rabbits and sheep, cows also eat grass. Worse yet, what happens if there are also foxes on the range? Competition between rabbits and sheep is not the same as between rabbits and foxes. Faced with a finite amount of grass, sheep would probably lament at the rapid multiplication of rabbits, whereas foxes would undoubtedly rejoice.

The main feature of the Volterra-Lotka equations is that they can deal with how one competitor influences the growth rate of the other. They do this by introducing a third parameter, the so-called coupling parameter. Sheep and rabbits have a negative effect on each other's population because they reduce each other's food supply. In contrast, foxes damage rabbit populations, while rabbits enhance fox populations. The coupling parameter reflects how much one species affects another — in other words, how many sales you will lose or win because your competitor won one. The magnitude of the parameter measures your ability to attack, counterattack, or retreat.

Advertising Strategies

The Volterra-Lotka model has three parameters for each competitor — one reflecting the competitor's ability to multiply, the second the size of its niche, and the third the interference from the other competitor. Thus, there are three lines of marketing action, or six if we also consider the parameters of the other competitor, see Table 2. To increase our prospects for growth, we can try to influence one or more of the following:

- the product's attractiveness (increase ours or decrease theirs),
- the size of the market niche (increase ours or decrease theirs), and
- the nature of the interaction (increase our attack or decrease their defense).

Each line of action affects one parameter at a time, but it is not obvious which change will produce the greater effect at a given time or which parameter is easiest to change. It depends on the particular situation. The concrete actions may include performance improvements, price changes, image transformation, and advertising campaigns. Performance and price concern "our" products only, but advertising with an appropriate message can in principle influence all aspects of competition, producing an effect on all six parameters. The question is how much of an effect a certain effort (budget) will produce.

Table 2. Six basic advertising strategies are defined by increasing or decreasing the parameters: attractiveness, niche size, and competition.

	ATTRACTIVENES	NICHE SIZE	COMPETITION
WE	Our products are good	You need our products	We are different
THEY	Their products are not good	You do not need their products	What they do, we do better

Some advertising messages have proven significantly more effective than others. Success is not necessarily due to whim, chance, or other after-the-fact explanations based on psychological or circumstantial arguments. The roles and positions of the competitors determine which advertising message will be most effective. Actual messages are often elaborate, but in principle, all successful advertising campaigns have exploited some combination of these six elements (Modis, 2003).

Carpet Wars

The effectiveness of advertising messages can be illustrated by a classical competitive technological substitution, that of synthetic fiber for natural fiber in the fabrication of carpets. For centuries, carpets were woven on a loom for which wool was well suited. But around the middle of the 20th century, a new tufting technique favored long, continuous filaments. At the same time, synthetic fibers such as nylon became available, and nylon-tufted carpets began replacing woven-wool rugs.

Solving the Volterra-Lotka equations for the carpet-sales data yields negative coupling constants for the two competitors, a typical situation of pure competition of the rabbit-sheep type. But the attacker's advantage was greater than the defender's counterattack, and so was its attractiveness. Therefore, the fate of the defender was eventual extinction. Today, woven-wool carpets represent less than 1% of carpet sales.

Could the makers of woven-wool carpets have secured a market niche the way fountain pens did? If so, what line of action should have they adopted? We can go back to 1979 and play out six scenarios exploring alternative lines of advertising — changing the six parameters one at a time by the same amount — to test their results. It turns out that effective campaigns would have been those that emphasized attractiveness and differentiation with messages such as "Wool is good" and "Wool is different from nylon" as opposed to a counterattack along the lines: "Wool is better than nylon." These conclusions could not have been arrived at by intuitive or other methods traditionally used by advertising agencies, and they could be completely different at another time or in another market (Modis, 2003).

Of crucial importance, of course, is the amount of effort required to achieve the targeted change. There is a way to estimate the size of the advertising investment needed. An advertising campaign along the line "Our product is good" affects the product's attractiveness just as a price cut does. The costs incurred from price dropping can thus be compared to those of an advertising campaign that achieves the same result. It should be noted, however, that if the survival of woolen carpets depended on price dropping alone, the price would have to be reduced to zero.

Effective Advertising

The Volterra–Lotka model accounts for the three fundamental factors that shape growth: the attractiveness of an offering, the size of its market niche, and its interaction with the competitor. When there is more than one competitor, the situation can be reduced to two by considering the major competitor only and by grouping all others together. Naturally, other factors influence growth, such as sales channels, distribution, market fragmentation, total market growth, market share, frequency of innovations, productivity, and organizational and human-resource issues. Many factors can be expressed as combinations of the three fundamental ones. Alternatively, the model could be elaborated — by adding more parameters — to take more phenomena into account.

As it stands, the model provides the baseline — the trend on top of which other, higher-order effects will be superimposed. It guides strategists through effective manipulations of a competitor's roles in the marketplace. It should be used before any discussions of investments, advertising tactics, or detailed planning take place. The model works equally well for products, for corporations, technologies, and whole industries. Only the time frames differ. Strategists now have a quantitative, science-based way to understand the crux of the competitive dynamics and to anticipate the consequences of possible actions.

A typical first question is, “Should we differentiate or counterattack?” You can answer this question with a simulation on a computer using sales data and the Volterra-Lotka equations. Just think — at this very moment there may be a cost effective way to terminate the state of being prey to the voracious competitor that has been feeding persistently on your achievements.

COMPETITION FOR NOBEL PRIZES

It has been suggested that the competition for Nobel Prize awards can be described by logistic-growth curves (Marchetti, 1989). The reasoning behind it was that the limited resource was the total number of Nobel laureates that the US will ever claim. The implication was that this number is capped. In other words, there will be some time in the future when all Nobel Prizes will be awarded to nationals from other countries. Up to that time, Americans would be elbowing each other to win prizes and the fewer left in their “niche” the harder it would be to win one.

My first attempt to fit a logistic S-curve to the cumulative number of US Nobel laureates in 1988 concluded that the US Nobel niche was already more than half full and implied a diminishing annual number of Nobel Prizes for Americans from then onward (Modis, 1988). Ten years later I confronted those forecasts with more recent data in my book *Predictions – 10 Years Later* (Modis, 2002). The agreement was not very good. The forecasts fell below the actual data and despite the fact that there was agreement within the uncertainties expected for a 90% confidence level the discrepancy did not go unnoticed. A technical note published in the same journal in 2004 highlighted the inaccuracy of my forecasts and cast doubt in the use of logistics to forecast US Nobel laureates (Golden & Zantek, 2004). On my part, I refit the updated data sample with a new logistic pointing to a higher ceiling and began wondering whether there was evidence here for the known bias of logistic fits to underestimate the final niche size. The new forecast again indicated an imminent decline in the annual number of American Nobel laureates.

Years later while preparing a new edition for my book — *Predictions – 20 Years Later* — I once again confronted forecasts with data. The situation turned out to be the same as ten years earlier, namely the forecasts again underestimated reality and despite agreement with the result of ten years earlier within the uncertainties expected for a 90% confidence level there was now clear disagreement between recent actual numbers and the original forecasts of twenty years earlier. The situation was reminiscent of the celebrated Michele-parameter episode in experimental physics where a measurement repeated many times over the period of fifty years kept reporting an ever-increasing value always compatible with the previous measurement but finally ending up in violent disagreement with the very first measurement.

So I wanted to settle the question of the ever-growing ceiling of the logistic curve fitted to the US Nobel laureates once and for all. One explanation for the ceiling of the S-curve to be constantly increasing is the fact that the US population itself has also been increasing over the same historical period. An increasing population provides an increasing “niche” for Nobel

Prize winners. But in order to solve the logistic equation the size of the ceiling must remain constant throughout the growth process.

An obvious way to account for the growing American population would be to study the number of laureates per capita thus rendering the ceiling of the S-curve time-independent. When I repeated the previous analysis for Nobel Laureates normalized to population I obtained better fits and consistency, namely the S-curves for all three time periods had ceilings that agreed within the expected uncertainties from each other.

Yet, there was still some tendency for the ceiling to increase with time, which suggested that considering US Nobel laureates per capita did not fully account for the increase of the "niche" size over time. In fact, the niche of individuals qualified for Nobel-Prize candidature in America could be increasing faster than the average population. After all, in my study I classified laureates with double nationality as nationals of the nation where the research for which they were being distinguished was accomplished. America, as a rule, welcomes research scientists from all over the world while it thwarts immigration by the uneducated. It could very well be that the population sample capable of producing Nobel laureates in America is growing faster than the rest of the population. Also I obtained fits of decreasing quality in longer data sets, and counterintuitive forecasts for a dramatic decline of American Nobel laureates and/or a major increase of the American population by the second half of the 21st century. So there was a need for deeper understanding of the Nobel-Prize competition.

A Bigger Picture

Besides the competition among Americans there is also competition between Americans and nationals of other countries. To the extent that US Nobel laureates represent about half or more of all Nobel Prizes every year, it is a good approximation to consider a duopoly, namely a niche with only two species: Americans and all others grouped together. The species "all others" is rather inhomogeneous but with US Nobel laureates and all Nobel laureates both being well defined as species candidates, "all others" also becomes a well-defined species candidate.

The competition between two populations growing in the same niche renders itself for a description by the Volterra-Lotka system of equations mentioned earlier. A global fit to all the data turned out to be of acceptable quality. The results are graphed in Figure 7 and tabulated in Table 3. The American trajectory is S-shaped (but not a logistic S-curve) and the long-term forecast is roughly a 50-50 split of all Nobel Prizes between Americans and all other nationalities.

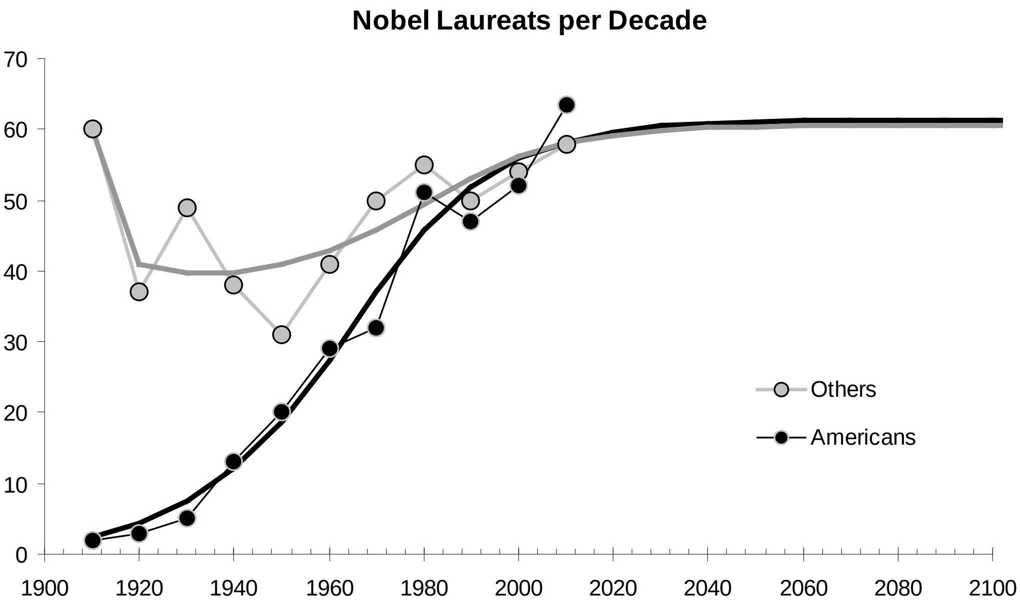

Figure 7. Decennial data points and solutions to the Volterra-Lotka equations. Despite its S-shaped form the black line is only approximately a logistic S-curve (Modis, 2010). This graph has been published in Modis (2010).

Table 3. Results for Volterra-Lotka Fits

	Attractiveness	Niche size	Competition
Americans	1.5	26	0.6
Others	1.7	37	0.4

Of particular interest are the values of the coupling constants in the Competition column of Table 3. They are both positive indicating a win-win nature for the competition. In a symbiotic relationship each competitor benefits from the existence of the other, which is in line with the dynamics of scholarly research (each publication triggers more publications). But Americans benefit more when non-Americans win Nobel Prizes than vice versa. A ratio 1.5 implies that one Nobel Prize won by a non-American will trigger 1.5 times more Nobel Prizes for Americans than the other way around. This is counteracted to some extent by the smaller attractiveness constant for the Americans.

The attractiveness constant reflects the species' ability to multiply. For product sales it indicates how many new sales will be triggered by one sale. In nature attractiveness represents the average litter size for a species (Modis, 1998). If it is greater than 1, the species population grows; if it is less than 1, it declines. The values in Table 3 shows attractiveness values for Americans and all others of 1.5 and 1.7 respectively. This means that each American Nobel laureate will "brood" 1.5 new American Nobel winners whereas for all others this number is 1.7.

All in all, the number of American Nobel Laureates in the long run should stabilize around an average of 61.4 per decade barely higher than 60.6 for all others.

Discussion

Competition arises when there are different entities vying for a limited resource. The two approaches considered here, i.e. logistic growth and Volterra-Lotka, correspond to different competitive struggles. In the first one the limited resource is the total number of Nobel laureates that the US will ever claim. The implication is that this number is capped. In other words, there will be a time when all Nobel Prizes will be awarded to nationals from other countries. Up to that time, Americans will be elbowing each other to win prizes and the fewer left in their “niche” the harder it will be to win one.

Logistic growth descriptions have been successful when used with products filling their market niche, epidemics filling their niche of victims, and in general each time a niche is filled or emptied in competitive circumstances. The approach renders itself for fitting an S-curve on cumulated historical data.

In the second approach — Volterra-Lotka equations — the competition with another species is also taken into account. The niche now is all Nobel Prizes awarded annually, not only the ones destined for Americans. This competitive struggle can take many forms the most publicized of which is the predator-prey struggle in which the predator grows on expense of the prey but also depends on the prey so that when the latter diminishes in numbers the predator also diminishes and oscillations ensue. But with Nobel Prizes no oscillatory behavior is observed. The competitive struggle turns out to be a win-win relationship and following some substitution in the early 20th century the two species grow in parallel to a peaceful and stable coexistence in a symbiotic relationship.

Interestingly the US trajectory is S-shaped, which suggests that a logistic fit could have been a reasonable approximation but not on the cumulative numbers. The fit should have been on the numbers per unit of time. The limiting resource in this case would have been the annual number of American laureates. This number was zero at the turn of the 20th century and progressively grew to 8 by 2009 (6.4 on the average during the nine years 2001-2009). The meaning of competition in this picture would be that Americans elbow each other every year for one of their “quota” prizes that grew along an S-curve and in the 21st century reached a ceiling of 6.1.

Recapitulating

Logistic S-curves are special cases of solutions to the Volterra-Lotka system of equations. The Volterra-Lotka Equations reduce to the logistic Equation whenever the coupling constants become null. Whereas logistic growth describes competition only among the members of one species, the Volterra-Lotka system of equations handles competition also with other species. It is advisable to consult the Volterra-Lotka approach — whenever possible — even if one is interested only in logistic growth because it can shed light on how to apply the logistic-growth equation. In the US Nobel-laureates study the Volterra-Lotka solution dictates that a logistic S-curve should be fitted on the annual numbers and not on cumulative numbers. Had we done so we would have obtained an answer very close to the black S-shaped curve of Figure 7.

Deciding whether to fit S-curves on cumulative or on per-unit-of-time data is a crucial first step for all logistic-growth applications and constitutes treacherous terrain for inexperienced S-curve enthusiasts. I myself mastered it only later in my career (Modis, 2007).

The forecasts for American Nobel laureates from the Volterra-Lotka approach are stable around an annual average of 6.1, comparable to the number of Nobel laureates won by all other nationalities together. Moreover the fitted parameters give rise to some interesting insights. The competition between Americans and all others for Nobel Prizes is of the win-win type. Locked in a symbiotic relationship both sides are winning but Americans are profiting more by 50%. At the same time, the ability of Nobel laureates to “multiply”, i.e. the extent to which a Nobel laureate incubates more laureates, is lower for Americans than it is for other nationalities. One may ponder whether the roots of this last observation have something to do with the fact that chauvinistic traits tend to be more endemic in cultures with longer traditions.

All conclusions need to be interpreted within the uncertainties involved. The quality of the logistic fits worsens as the time window increases. Normalizing to the population improves the quality of the fits. A confidence level of 72% indicates that there is 7 out of 10 chances that the Volterra-Lotka description is the right way to analyze this competition, not very different from the S-curve fit on the data normalized to population. For the intermediate future — ten to twenty years — the logistic normalized to reasonable population projections would result in forecasts compatible with those of the Volterra-Lotka approach. Still, I would choose Volterra-Lotka because it addresses a more general type of competition. In any case, long-term forecasts cannot be reliable and the whole exercise must be repeated with updated data sets in a couple of decades, by which time it may be appropriate to consider more than just two players.

CONCLUSION

The natural law describing growth in competition has a very simple formulation, the logistic equation. The simpler a law the more fundamental it is and the wider its range of applications. The successful scientific description of competition in the cases considered in this chapter yields unexpected insights that include:

- All natural growth is capped. Sustained growth can only take place in successive well-defined natural-growth steps.
- Demystification of popular wisdom such as “easy come, easy go” and “you need gold to make gold.”
- The indispensable role of teachers and the inherent difficulty in every beginning.
- Car fatal accidents are maintained at a “desirable” level by society as a concomitant of things of greater value.
- Death correlates with the end of one’s productivity/creativity; Mozart may have died exhausted of creative musical potential.
- There is no universally best way to compete; the appropriateness of counterattack, cooperation, or differentiation depends on the particular situation.

- Americans are likely to continue dominating Nobel Prizes awards because they better leverage the win-win relationship they enjoy with Nobel Prizes won by non-Americans.
- The extent to which a Nobel laureate incubates more laureates, is lower for Americans than it is for other nationalities. One may ponder whether the roots of this have something to do with the fact that chauvinistic traits tend to be more endemic in cultures with longer traditions.

Europeans were first to reach the New World thanks to their physical proximity and their mastering of the maritime and navigation sciences. But the transfer of Nobel awards from Europeans to Americans during the first half of the 20th century makes one wonder whether the blossoming of the New World was not due to a competitive advantage stemming from a science-based culture.

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APPENDIX – THE MATHEMATICAL FORMULATIONS BEHIND NATURAL GROWTH IN COMPETITION

A1. The Logistic and the Chaos Equations

Logistic growth describes how a species population grows to fill its ecological niche under conditions of natural competition (survival of the fittest). The same law also describes how we learn, and how rumors and epidemic diseases spread. The law is cast in the following differential equation:

$$\frac{dX}{dt} = aX(1 - X) \text{ where } a \text{ is a constant} \qquad (1)$$

But when Equation (1) is put in the form of a difference equation, it becomes

$$X_{n+1} = r\, X_n(1 - X_n) \text{ where } r \text{ is a constant} \qquad (2)$$

This equation is strikingly similar to Equation (1), but whereas Equation (1) gives rise to the smooth S-shaped logistic pattern, Equation (2) for certain values of r gives rise to states of chaos. The former emphasizes the presence of a trend and has become the tool to describe natural growth. The latter emphasizes the lack of trend and has become the tool to describe chaos. The chaotic fluctuations appear on what corresponds to the ceiling of the logistic after the upward trend has died down.

A2. The Solutions

The solution of Equation 1 (obtained by integration) is given below; it yields the S-shaped pattern of the S-curve.

$$X(t) = \frac{M}{1 + e^{-a(t-to)}}$$ where t_o is an integration constant

The solution of Equation 2 (obtained by iteration) for $r > 3.7$ gives the following pattern: